FUZZY LOGIC SYSTEMS

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ABSTRACT: This is one of a series of white papers on systems modelling, analysis and control, prepared by Control Systems Principles.co.uk to give insights into important principles and processes in control. In control systems there are a number of generic systems and methods which are encountered in all areas of industry and technology. These white papers aim to explain these important systems and methods in straightforward terms. The white papers describe what makes a particular type of system/method important, how it works and then demonstrates how to control it. The control demonstrations are performed using models of real systems designed by our founder - Peter Wellstead, and developed for manufacture by TQ Education and Training Ltd in their CE range of equipment. Where possible, results from the real system are shown. This white paper is about the very simple, but very useful method of fuzzy logic and fuzzy control.

1. Why Fuzzy Logic?

Normally in logic we have a series of statements which are either true or false, yes or no, 0 or 1. In this context, the statement ‘the temperature is 25 degrees Celsius’ is an objective one and is either true or false. However, for many situations the answer is more like ‘Errr’ – ‘not sure’ – ‘maybe’ – ‘that depends’ and so on. For example, on a pleasant summer’s day the statement ‘the temperature is too high’ is neither true nor false. The statement is a qualitative one – it represents an opinion rather than an objective fact. For example, it needs to be a bright sunny day on the beach for me to feel warm. On the other hand, I could mention some visiting scientists at Control Systems Principles who feel comfortable in a snow storm on top of a mountain. Do you see what I mean? There is no certainty to the situation – it depends upon the context.

Fuzzy logic deals with uncertainty in engineering by attaching degrees of certainty to the answer to a logical question. Why should this be useful? The answer is commercial and practical. Commercially, fuzzy logic has been used with great success to control machines and consumer products. In the right applications fuzzy logic systems are simple to design, and can be understood and implemented by non-specialists in control theory. In most cases someone with a intermediate technical background can design a fuzzy logic controller. The control system will not be optimal but it can be acceptable. Control engineers also use it in applications where the on-board computing is very limited and adequate control is enough. Fuzzy logic is not the answer to all technical problems, but for control problems where simplicity and speed of implementation is important then fuzzy logic is a strong candidate. A cross section of applications that have successfully used fuzzy control includes:

Environmental Control

- Air Conditioners
- Humidifiers

Domestic Goods

- Washing Machines/Dryers
- Vacuum Cleaners
- Toasters
- Microwave Ovens
- Refrigerators

Consumer Electronics

- Television
- Photocopiers
- Still and Video Cameras – Auto-focus, Exposure and Anti-Shake
- Hi-Fi Systems

Automotive Systems

- Vehicle Climate Control
- Automatic Gearboxes
- Four – Wheel Steering
- Seat/Mirror Control Systems

This is an impressive list, and gives an idea of the key application areas. In general you will not find a fuzzy controller in a safety critical application, unless the practical and theoretical performance has been completely studied.

2. Engineering Motivation

A traditional logic decision block produces an outcome based upon binary logic. A firm YES or NO emerges as an output of the decision block. However, the inventor of fuzzy logic, Lofi Zadeh, noted that human decision making incorporates shades of meaning in which the binary YES/NO might be replaced by:

DEFINITELY YES,
PROBABLY YES,
MAYBE,
PROBABLY NO,
DEFINITELY NO.

Fuzzy logic copies this feature of human decision making using levels of possibility in a number of uncertain (or fuzzy) categories. For example, think about the Coupled Tanks System (See Elke's white paper on Coupled Tanks Systems on the download page of www.control-systems-principles.co.uk for full details) in which the object is to adjust the input voltage, u, to the pump motor (Figure 1) so that the level in Tank 2 is held at a steady value. The measured output is the level in Tank 2, denoted by the signal y_2. My colleague Elke would apply a PI controller to this system as fast you could say 'the hills are alive with the Sound of Music'. But if the exact level is not important then why not use a simpler controller? For example, a common sense controller could use the following fuzzy control rules:

IF {level too high} THEN {reduce pump voltage}

IF {level too low} THEN {increase pump voltage}

IF {level correct} THEN {set pump voltage zero}

The controller performance would not be as good as a PI controller, but it might be acceptable – and that is what we are after – the simplest and cheapest possible controller for a given application. Sorry about that Elke.

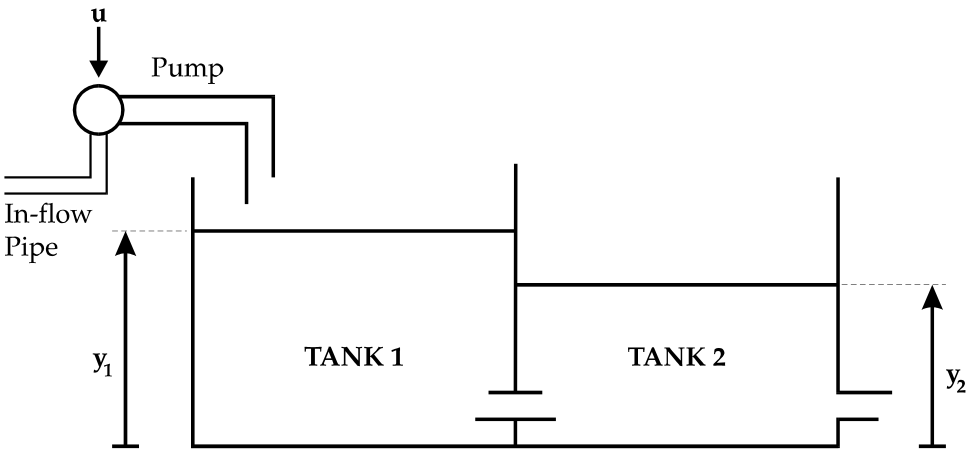

Figure 1. Coupled Tanks System

3. How to Do Fuzzy Logic

3.1. Classification - Turning a Real Signal into a Set of Fuzzy Variables

The first step in fuzzy logic is to convert the measured signal x (which might be the error signal in a control system) into a set of fuzzy variables. This is called **fuzzy classification** or **fuzzification**. It is done by giving values (these will be our fuzzy variables) to each of a set of membership functions. The values for each membership function are labelled $\mu(x)$, and are determined by the original measured signal x and the shapes of the membership functions. A common fuzzy classifier splits the signal x into five fuzzy levels as follows:-

a) LP: x is large positive
b) MP: x is medium positive
c) S: x is small
d) MN: x is medium negative
e) LN: x is large negative

Membership functions for three of these five fuzzy levels are shown in Figures 2a. So, for example, the value (or fuzzy variable) for the MP membership function and a signal value of $x = 2.5\text{v}$ is $\mu_{mp}(2.5) = 0.5$.

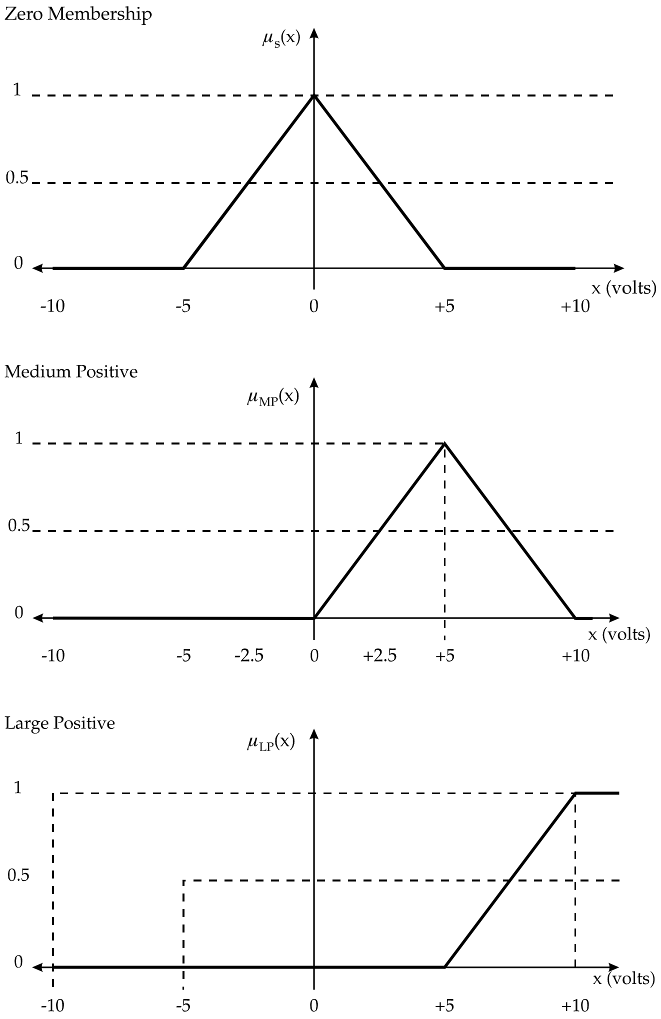

Figure 2a. Membership Functions for Zero Membership (S: x is small), Medium Positive Membership (MP), Large Positive Membership (LP).

Figure 2a only shows the zero (or small S), medium positive (MP), and large positive (LP) memberships. The remaining two (MN, LN) are the same as the MP and LP shapes, but with the x axis reversed. Figure 2b shows all five membership functions on the same axis.

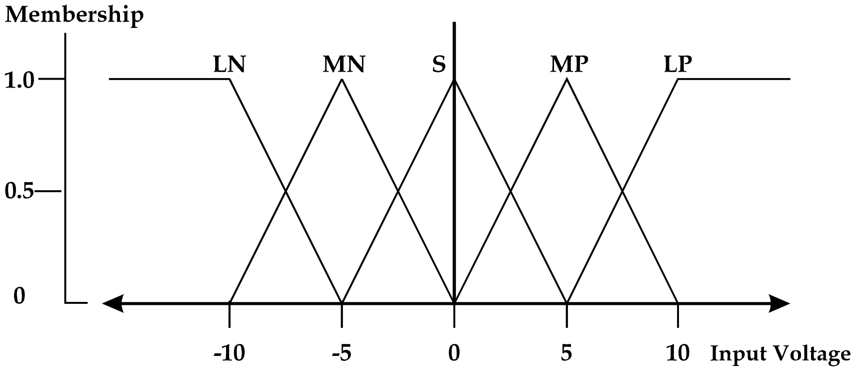

Figure 2b. The Complete Set of Membership Functions for Five Level Fuzzification

The shape of the membership functions in Figures 2a and 2b is termed 'triangular' – this is only one of many choices of membership function shapes. I use triangular shapes because they are widely used, simple to implement and give good results.

A practical fuzzifier would have a measured signal from a sensor at its input and would provide at its output the values (fuzzy variables) corresponding to the membership functions. For example, if a sensor signal with an output voltage of 2V is applied to a five level fuzzifier, the resulting set of fuzzy variables is:

$$\mu_{\mathrm{LN}} = 0$$
$$\mu_{MN} = 0$$
$$\mu_S = 0.6$$
$$\mu_{MP} = 0.4$$
$$\mu_{LP} = 0$$

As the input to the fuzzifier changes in the range –10v to +10v, then the corresponding fuzzy variables will also change.

In a controller the fuzzifier is used to determine the level of membership by connecting a measured signal from the system to the fuzzifier input. For example, if the five level fuzzifier is connected to the Coupled Tanks System, then the membership value associated with a statement like: *"the level in Tank 2 is large positive"* is obtained by connecting the signal for level in Tank 2 (y_2) to the input of the fuzzifier and monitoring the LP output of the classifier as in Figure 3.

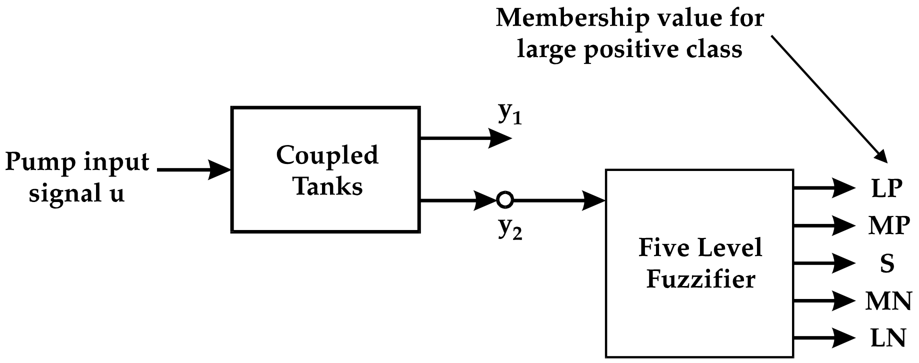

Figure 3. Measuring the Membership Value 'Level 2 is Large and Positive'.

3.2. Fuzzy Decisions Blocks

Fuzzy control uses fuzzy equivalents of logical AND, OR and NOT operations to build up fuzzy logic rules. The definitions of these are:

AND: If μ_{α} is the membership of class α for a measured variable μ_{β} and is the membership of class β for another measured variable, then the fuzzy AND is obtained as the *minimum* of the two membership values:

$$\mu_{\alpha \,\mathrm{AND}\, \beta} = \mu_{\alpha} \wedge \eta_{\beta} = \min(\mu_{\alpha}, \mu_{\beta})$$

Where the symbol $\wedge$ is used to denote the fuzzy AND operation. An alternative definition of the fuzzy AND is that is the *product* of the two membership values:

$$\mu_{\alpha \,\mathrm{AND}\, \beta} = \mu_{\alpha} \wedge \mu_{\beta} = \mu_{\alpha} \times \mu_{\beta}$$

We have used both in practical applications and there is not much difference, so I generally recommend the first definition of fuzzy AND.

OR: The previously given definitions of $\mu_{\alpha}\mu_{\beta}$ apply again, and the fuzzy OR function is defined as:

$$\mu_{\alpha OR \beta} = \mu_{\alpha} \vee \mu_{\beta} = \max(\mu_{\beta}, \mu_{\beta})$$

NOT: For membership μ_{α} the fuzzy NOT operation is defined by:

$$\mu_{\mathrm{NOT}\alpha} = \neg \mu_{\alpha} = 1 - \mu_{\alpha}$$

Where the symbol $\neg$ is used to denote the fuzzy NOT operation.

3.3. Defuzzification - Turning a Set of Fuzzy Variables into a Real Signal

The last step in building a fuzzy logic system is turning the fuzzy variables generated by the fuzzy logic rules into a real signal again. The fuzzy logic process which does this is called **defuzzification** because it combines the fuzzy variables to give a corresponding real (crisp or non-fuzzy) signal which can then be used to perform some action. For example, in the case of a coupled tanks control system the crisp signal would be a voltage which can be used to actuate the pump drive amplifier.

A five level defuzzifier block (Figure 4) will have inputs corresponding to the following five actions:

a) LP: Output signal large (positive)
b) MP: Output medium (positive)
c) S: Output signal small
d) MN: Output signal medium (negative)
e) LN: Output signal large (negative)

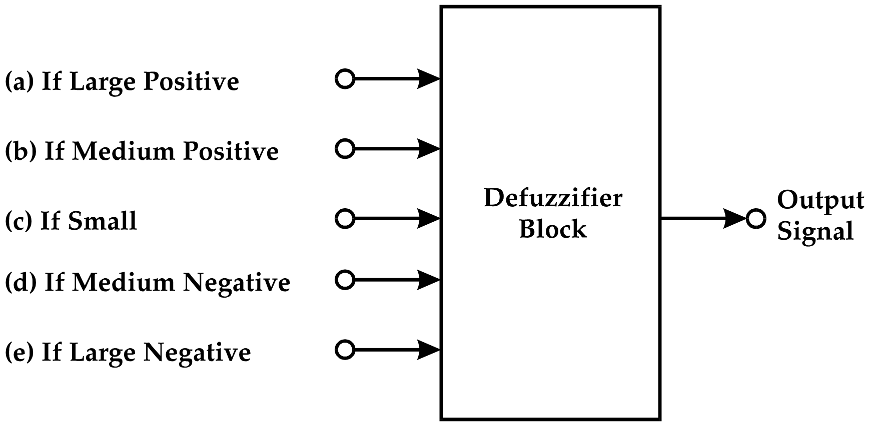

Figure 4. Block Diagram of a Defuzzifier

The defuzzifier combines the information in the fuzzy inputs to obtain a single crisp (non-fuzzy) output variable. There are a number of ways of doing. This is the simplest and most widely used method and is called the centre of Gravity Method. It works as like this: If the fuzzy levels LP......LN have membership values that are labelled μ_1.....μ_5, then the crisp output signal u is defined as:-

$$u = \frac{\sum_{i=1}^{5} u_i \mu_i}{\sum_{i=1}^{5} \mu_i}$$

Where the values of the u_i are, $u_1 = 10V, u_2 = 5V, u_3 = 0V, u_4 = -5V, u_5 = -10V$, and correspond to the central points of the fuzzy classes LP: MP: S: MN: LN at the input to the defuzzifier. Defuzzifier input terminals which have no connections have fuzzy input values of zero.

4. Developing Fuzzy Logic Control Rules

Many research papers have been written on how to create fuzzy rule sets. Most of these methods are mathematical and require analytical knowledge to understand them. In our view this defeats the purpose of fuzzy logic. The main motivation for fuzzy logic is that by simply writing down common sense rules it is possible to build a reasonable control strategy without deep theoretical knowledge of control. This means that we will have no knowledge of the stability properties of the controller, and so the scope of applications is restricted to fairly simple control applications. This is fine because there are simple control problems that just want a simple solution. I have mentioned already the domestic products market, but we can add to this some of the simpler industrial control loops.

A fuzzy control system is obtained by writing a set of rules of the form:

IF {situation} THEN {action}

The procedure is to write down the basis rules and add and refine them based upon experience. In the example of the coupled tanks system, a fuzzy rule which forms part of a control system might be:

IF {error small} AND {control signal large positive}

THEN {control signal small} **(#1)**

The fuzzy levels 'error small' and 'control signal large positive' would be obtained by using the error, **e**, and control signal, **u**, as input signals to two separate five level fuzzifiers and selecting outputs S and LP respectively. The fuzzy levels are then fed through a fuzzy AND block to obtain a fuzzy value that gives the membership value corresponding to the situation. A fuzzy system which implements the fuzzy control rule **#1** is shown in Figure 5.

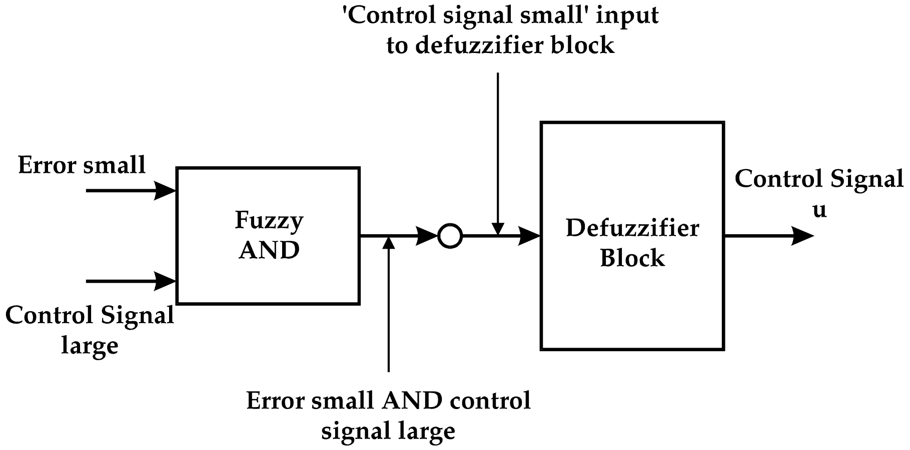

Figure 5. An Implementation of Fuzzy Rule #1

In most fuzzy logic control systems the set of fuzzy actions is a simple list of rules for an open loop sequence. An example of an open loop fuzzy control sequence is that which might be used in a washing machine water heater control where water temperature is not really critical. In a simplified version of a wash cycle the washing machine design engineer may wish to begin the wash with maximum heater power and gradually reduce the power as the wash-cycle time increases, ending with zero power at the completion of the wash cycle. A fuzzy rule set for this control cycle is:-

IF {cycle-time small} THEN {heater power large positive}

IF {cycle-time medium positive} THEN {heater power medium positive}

IF {cycle-time large positive} THEN {heater power small}

Notice that the rules in this fuzzy control sequence are such that they can be simply written down and sequentially implemented in software using the fuzzy blocks described previously. The fuzzy rule set becomes more complex if the fuzzy level state associated with more than one signal must be accounted for. In such cases the rules contain interactions such that the fuzzy AND/OR of two or more fuzzy variables might be required. For example, consider again the washing machine control cycle in which the designer wants to become more sophisticated by taking account of the water temperature. Specifically, suppose that the designer wants to prevent the wash temperature becoming high, then the third rule could be changed to:

IF {cycle-time large positive} OR {water temperature large positive} THEN {heater power small}

The use of the fuzzy OR means that a more complex control cycle is created in which the heater power is reduced if the temperature gets too high as well as at the end of the wash.

From the above simple example, you can see how we put together fuzzy rules and add to them as and when new needs appear. In some cases the set of fuzzy rules can get long and hard to follow. In these cases fuzzy state tables are sometimes used. These are logic tables that enable a large set of fuzzy AND statements to be compressed into simple tabular form. The CE124 Fuzzy Logic System which is described below, uses a state table for this reason.

5. Fuzzy Control in Feedback Systems

The most suitable applications for fuzzy control are where there are qualitative requirements for a satisfactory control action and these qualitative requirements can be easily stated as fuzzy logic rules. For this reason, fuzzy logic controllers are used to operate the automatic functions of washing machines, video recorders, compact disk players, air conditioning systems, cameras and similar products. It is also possible to find fuzzy logic in industrial feedback control that is normally done by experienced human operators who have manual control over a complex process. The procedure followed is to put the operator's control procedure into a fuzzy rule set and hence develop a fuzzy control system. The fuzzy logic designer copies the heuristic actions of a human operator when they control a process and writes down the corresponding fuzzy rule. By careful observations of a skilled operator, a complete set of fuzzy rules is obtained which can reproduce the best performance of the human operator. The result is an "intelligent" control system that is obtained without reference to control systems theory but contains the knowledge of a good human operator. There was great enthusiasm for this approach in the 1970 and 1980 period, but it was found that the human operator can not be easily replaced and now such fuzzy systems are used as an advisor to the operator – the idea is to improve safety rather than replace the operator.

A popular application of fuzzy logic is the control of simple loops usually controlled using three term (PID) controllers. The fuzzy logic copies the PID action with some modifications to handle non-linear plant behaviour. Figure 6 shows how a fuzzy logic system might replace a conventional controller.

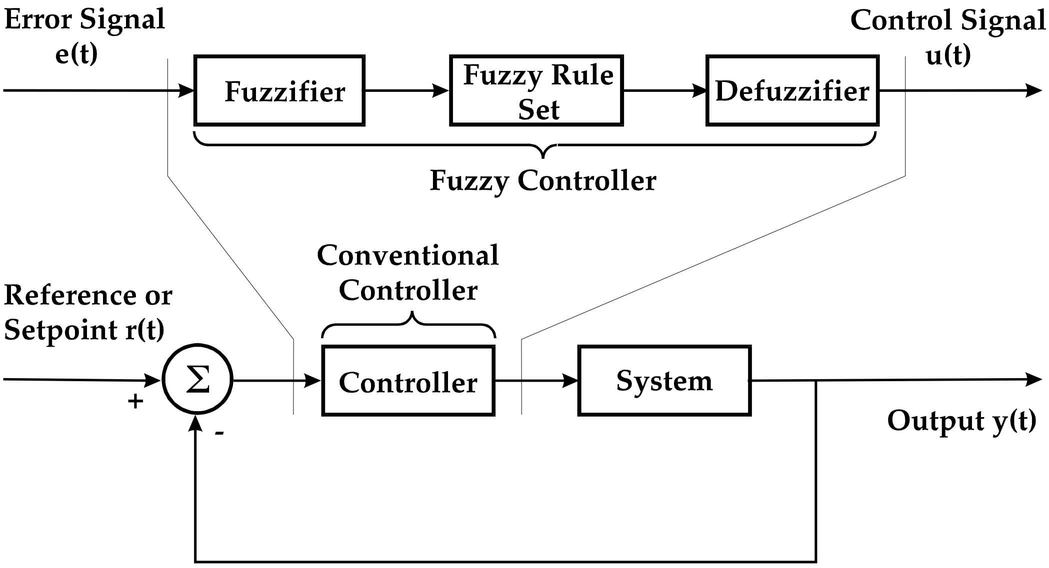

Figure 6. The Fuzzy Controller and its Relation to a Conventional Control Loop

The procedure adopted in fuzzy control is to imitate the actions of a traditional controller using fuzzy rules and add features to deal especially with special system features such as nonlinearities.

5.1. Fuzzy Proportional Control

A very simple fuzzy version of proportional controller is

Rule 1: IF {error LN} THEN {control LN}

Rule 2: IF {error MN} THEN {control MN}

Rule 3: IF {error S} THEN {control S}

Rule 4: IF {error MP} THEN {control MP}

Rule 5: IF {error LP} THEN {control LP}

In fact this rule set produces exactly the same linear control action as a controller with proportional control and a gain of one, operating on the error signal. This brings no advantage at all. However, the gain of the controller can be made non-linear by changing the fuzzy rules, and this can be useful in special applications. To give a simple example, consider the case of the Coupled Tanks System, where the input pump cannot be driven backwards to suck water out of the tank (e.g. the minimum control input is 0V) and the pump input signal amplifier will not accept more than a maximum of 10V. These practical limits on the working range of a control systems actuator are normal, and it is easy to adapt a fuzzy controller to account for them. A fuzzy proportional controller which incorporates the constraint that the minimum input signal is 0V is:-

Rule 1: IF {error LN} OR {error MN} OR {error S} THEN {control S}

Rule 2: IF {error MP} THEN {control MP}

Rule 3: IF {error LP} THEN {control LP}

Here the level S is 0v and Rule 1 ensures that the pump does not receive negative signals. Figure 7 is an implementation of this rule set

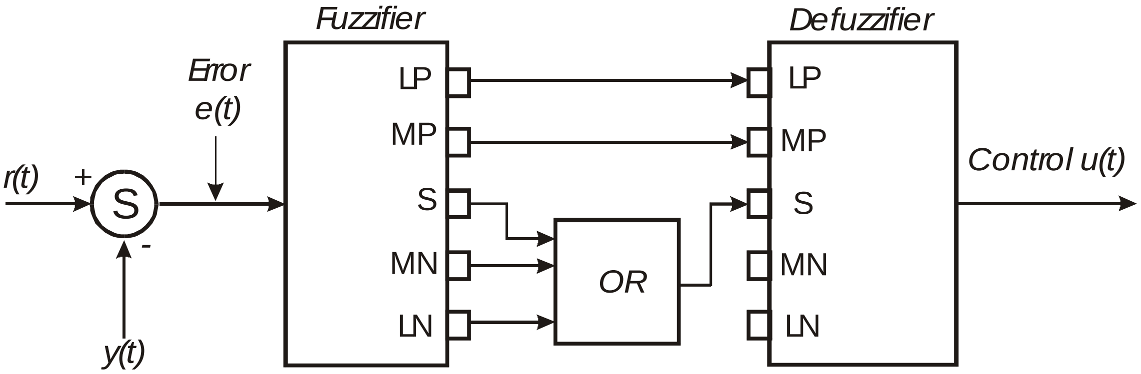

Figure 7. Implementation of Fuzzy Rule Set RS4

This is a simple demonstration. With more complex fuzzy rules complex non-linearities can be achieved. Many control professionals will question whether this is the correct way to design a non-linear controller, however practical engineers sometimes do this where they are sure of the stability implications.

5.2. Proportional Plus Derivative Fuzzy Control

The fuzzy proportional controller can be extended easily to cover integral action and derivative control. Here we outline just the derivative control extension. In this case the fuzzy logic controller operates on the error signal *e(t)* and the derivative of the output signal $\frac{dy(t)}{dt}$ and produces an output from its defuzzifier which is the control signal *u(t).* (See Elke's excellent white paper on three term control on the download page of www.control-systems-principles.co.uk to understand why the derivative of the output, and not the derivative of the error, is used). The fuzzy logic controller bases its actions on the two signals:- the error and the rate of change of output. In this context, it is important to notice that the fuzzy controller does not contain any dynamic elements. All dynamic components are outside the controller and are obtained by direct measurement from the system or by signal processing of the system input and output signals The output derivative is either available as a direct measurement from the system or by using an observer of the system states.

After fuzzification of the error and the output rate, fuzzy rules are applied to the fuzzifier variables. The role of rate feedback in a conventional controller is to reduce the control action if the output is changing too rapidly, this reduces the possibility of the output overshooting the desired reference value, r(t). Using this principle, fuzzy rules can be written which help to avoid such occurrences. For example, the rule set given below has the first five rules providing fuzzy proportional control. The Rules 6 and 7 try to compensate for rapid changes when the error is small by providing a component of the control which will decrease the rate of change of the system output.

Rule 1: IF {error LP} THEN {control LP}
Rule 2: IF {error MP} THEN {control MP}
Rule 3: IF {error S } THEN {control S}
Rule 4: IF {error MN} THEN {control MN}
Rule 5: IF {error LN} THEN {control LN}
Rule 6: IF {error S} AND {output_rate LP } THEN {control LN}
Rule 7: IF {error S} AND {output_rate LN } THEN {control LP}

This rule set approximates proportional plus derivative control action, but only when the error is small. By studying a conventional linear controller with rate feedback it is possible to form rule sets which imitate them. For example the conventional proportional plus derivative control law is:

$$u(t) = k_p e(t) - k_d \frac{dy(t)}{dt}$$

Where k_p is the proportional gain and k_d is the derivative feedback gain. From this equation it is possible to deduce a simple set of equivalent fuzzy rules:

Rule 1: IF {error N} AND {rate P} THEN {control N}
Rule 2: IF {error N} AND {rate N} THEN {control S}
Rule 3: IF {error P} AND {rate N} THEN {control P}
Rule 4: IF {error P} AND {rate P} THEN {control S}

The fuzzy control corresponding to this rule set is a very coarse approximation to the behaviour of a controller with proportional plus derivative action. With these four rules, the quality of control achieved would be poor. In order to see this, Figure 8 shows an isometric view of the control signal plotted as a function of the error and rate. Notice that the control action moves only between three levels +10V, 0V and -10V. The poor results of this rule set can be improved by adding more levels of fuzzification to achieve a closer approximation to the true control law. For example, Figure 9 shows the control surface corresponding to five levels of fuzzification in the control rule.

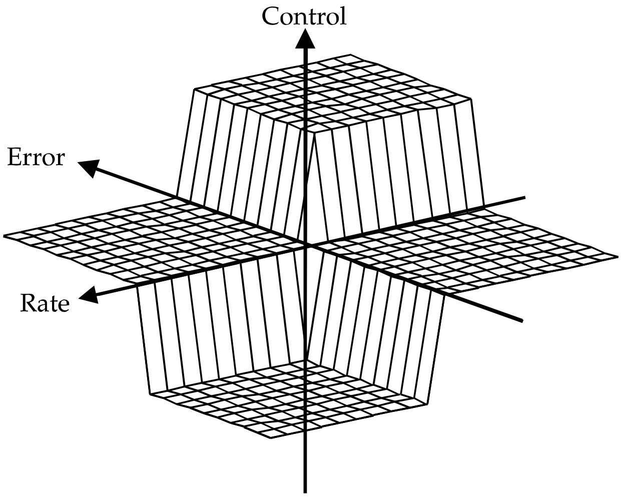

Figure 8. Isometric Plot of Control-Rate Signal for a Three Level Fuzzy Controller

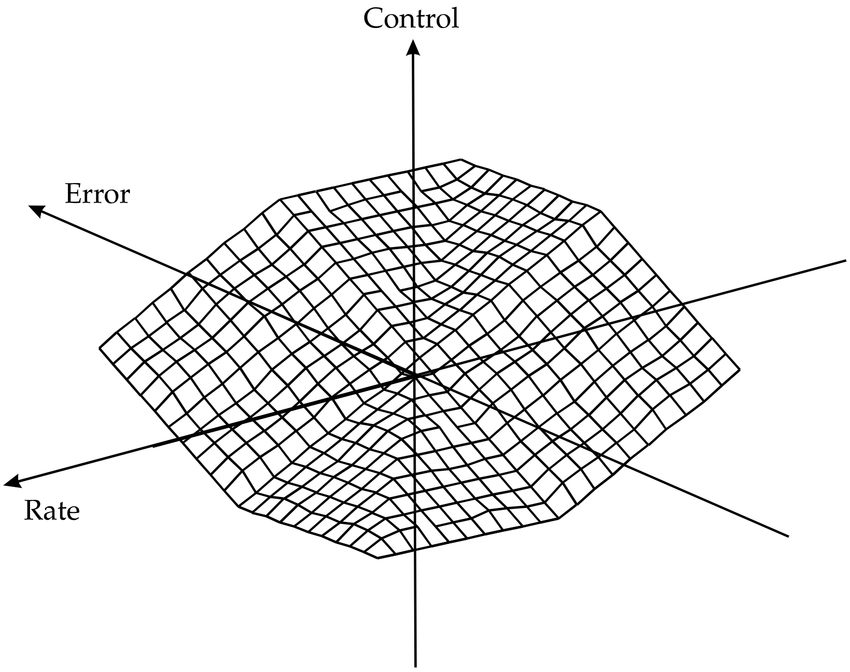

Figure 9. Isometric Plot of Control-Rate Signal for a Five Level Fuzzy Controller

The addition of the extra levels of classification give a much smoother control surface and by increasing the number of fuzzification levels yet further, a much closer approximation can be obtained. However it is not the purpose of fuzzy control to emulate conventional control – and I only include these results as a demonstration. Fuzzy control is at its best when used on simple systems (e.g. the domestic products I that have I mentioned earlier in this white paper), or where a nonlinear feature of the system can be compensated for easily in a set of special fuzzy control rules.

6. Fuzzy Logic Systems.

There are a number of fuzzy logic systems available, and the fuzzy logic system shown in Figure 10 is typical of these systems. It is laid out as a schemmatic of a fuzzy system would be, with two fuzzifiers on the left of the panel (above and below the TQ logo), a fuzzy state table at the top centre, and set of fuzzy logic AND, OR and NOT blocks (again in the centre), and a defuzzifier at the right hand side.

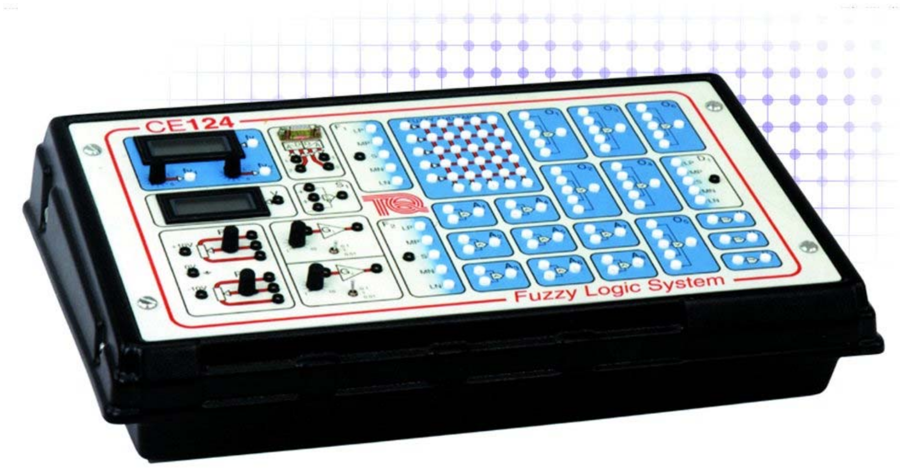

Figure 10. The CE124 Fuzzy Logic System

The system shown in Figure 10 is especially designed to allow users to quickly implement fuzzy logic systems in an intuitive way. Because the fuzzy logic components are separately represented, it is easy to connect up fuzzy rules from a block diagram and then to track the variations in fuzzy signals (using built in ‘fuzzy meters’) through the logic system. This is very important and useful during development and testing on real hardware. Specifically, the developer can measure the ways in which the membership functions vary over time and so check that fuzzy rules are actually doing what they are meant to do. Rewiring the fuzzy logic system is quick and easy, so that a fuzzy control system can be rapidly tested and improved on-line. For the same reasons, the CE124 is also a good learning tool. I have found that people who initially find fuzzy logic concepts difficult are able to understand the technique after a session with the CE124 hardware.

The CE124 has hardware and software options for fuzzy logic development. This contrasts with the other main family of fuzzy logic systems that are software packages and often require Matlab and Matlab Toolboxes to operate. Software packages can be very powerful tools and contain many advanced fuzzy system features. They are however usually simulation tools so that practical implementation is harder to acheive

7. A Final Word

I hope that you have got some ideas about the background and uses of fuzzy control from this white paper. We get lots of inquires for help and and advice on projects and applications, but the usual disclaimer applies - I am sorry to say that it is not possible to answer general questions about the contents of our white papers, unless we have a contract with your organisation. Control Systems Principles is a ‘tight ship’ and we do not have spare time for this kind of good will help. For more information about fuzzy control and the CE124 Fuzzy Logic System go to the TQ Education and Training web site using the links on our web site www.control-systems-principles.co.uk or use the email info@tq.com. Process level control and the Coupled Tanks system is also the download page of the Control Systems Principles web site, so do check that out.

I have worked on practical fuzzy control systems for some time and this white paper contains the most useful parts of my ‘fuzzy know-how’. If you want to know more about fuzzy systems, then there are many detail research papers and text books on fuzzy control, but I recommend that you start with the original papers by Lofti Zadeh. Fuzzy logic has been a fashionable playground among many researchers, so be selective in your reading. On the other hand, Japanese engineers and researchers have taken fuzzy logic seriously – so fuzzy logic applications from Japanese companies should be read carefully as they always relevant and often informative.