COUPLED TANKS SYSTEMS 1

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ABSTRACT: This is one of a series of white papers on systems modelling, analysis and control, prepared by Control Systems Principles.co.uk to give insights into important principles and processes in control. In control systems there are a number of generic systems and methods which are encountered in all areas of industry and technology. These white papers aim to explain these important systems and methods in straightforward terms. The white papers describe what makes a particular type of system/method important, how it works and then demonstrates how to control it. The control demonstrations is performed using models of real systems designed by our founder - Peter Wellstead, and developed for manufacture by TQ Education and Training Ltd in their CE range of equipment. Where possible results from the real system are shown. This white paper is about the most common control problem in practical process systems - liquid level control. In addition to the process industries liquid level control is found in many other places, as you will read below. Liquid level control and modelling ideas will be demonstrated using our version of the standard textbook system – the Coupled Tanks System. The Coupled Tanks System has become a 'design classic' of control engineering teaching equipment and you can see it in many control laboratories across the world.

1. Why Coupled Tanks Systems?

The control of liquid level in tanks and flow between tanks is a basic problem in the process industries. The process industries require liquids to be pumped, stored in tanks, then pumped to another tank. Many times the liquids will be processed by chemical or mixing treatment in the tanks, but always the level of fluid in the tanks must be controlled, and the flow between tanks must be regulated. Often the tanks are so coupled together that the levels interact and this must also be controlled. Level and flow control in tanks are at the heart of all chemical engineering systems. But chemical engineering systems are also at the heart of our economies. Vital industries where liquid level and flow control are essential include:

- Petro-chemical industries.
- Paper making industries.
- Water treatment industries.

Our lives are governed by level and flow control systems. For example, medical physiology involves many fluid bio-control systems. Bio-systems in our body are there to control the rate that blood flows around our body. Other bio-systems control the pressure and levels of moisture and chemicals in our body. The water closet (WC) toilet in your appartment or house is also a liquid level control system. The swinging arm attached to the input valve of the WC water tank allows water to flow into the tank until the float rises to a point that closes the valve. This is a simple and effective level control system for water tanks. Although the WC toilet is now common, I am told that the first WC in my home village was in the *Herrenhaus*. It was a thing of great wonder. Visitors would admire the automatic refilling of the WC tank much more than the beauty of the house and our beautiful countryside!

From the amazing Silveretta *Hochalpenstrasse* you will see a gigantic coupled tanks system - the *Silveretta Stausee*. The *Silveretta Stausee* is 2034 metres high, I think this is the highest artifical lake in Europe. The *Silveretta Stausee* is coupled with the *Vermunt Stausee* at 1717 metres and with its hydro-electric system it is the largest and highest coupled tank systems in the world. Engineers who designed and built this are very proud of their acheivement and special information panels near the *Stausee* describe the project.

Tank level control systems are everywhere. All of our process industries, the human body and fluid handling systems depend upon tank level control systems. It is essential for control systems engineers to understand how tank control systems work and how the level control problem is solved. So pay special attention to what I write. One day you may have to work on a grand project like the *Silveretta Stausee* and Elke will not aways be here to help you. The alternative may be to spend your life repairing toilets and in this situation Elke will certainly not help you. Either way level control systems are involved.

2. Modeling the Coupled Tanks System

2.1. Single Tank Model

It is important to understand the mathematics of how the coupled tanks system behaves. This is system modelling and it is a very important part of control systems analysis. To be begin look at a single tank system in figure 1:

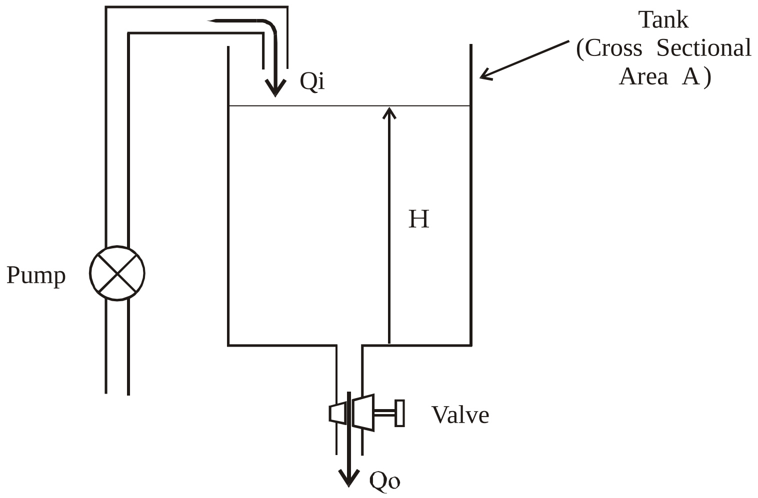

Figure 1: A Single Tank Fluid Level System

The system model is determined by relating the flow Q_i into the tank to the flow Q_o leaving through the Valve at the tank bottom. Using a balance of flows equation on the tank, it is possible to write:

$$Q_i - Q_o = A\frac{dH}{dt} \qquad \textbf{(1)}$$

where, A is the cross-sectional area of the tank, and H is the height of the fluid in the tank.

If the valve is assumed to behave like an idea sharp edged orifice, then the flow through the valve will be related to the fluid level in the tank, H, by the expression,

$$Q_o = C_d a \sqrt{2g.H} \qquad \textbf{(2)}$$

In this equation a is the cross sectional area of the orifice, (in practice the cross-sectional area will be given by the dimensions of the valve and the flow channel in which it is mounted). C_d is called the discharge coefficient of the valve. This coefficient takes into account all fluid characteristics, losses and irregularities in the system such that the two sides of the equation balance. And g = gravitational constant = 980 cm/sec^2.

Equation 2 assumes C_d is a constant so that Q_O has a nonlinear relationship to the level H for all possible operating conditions. Ideally the nonlinear relation is the square root equation 2, but in a practical valve there is a more complex non-linear equation. Combining Equations 1and 2 gives,

$$A\frac{dH}{dt} + C_d a\sqrt{2gH} = Q_i \qquad \textbf{(3)}$$

This equation is the mathematical model that describes the system behaviour, and again we see nonlinear things in the system model. Apparently it is just like my Servo Control white paper, but with an important difference. In the tank level problem the nonlinearity is smooth and can be made linear at a particular operating level H by using the slope of the nonlinearity at H. This has the important result that the linearized system model has parameters that depend upon the operating conditions. The system dynamics will change as the normal operating level changes. Please remember this, because it is very important that the tank level controller is insensitive to parameter changes in the model.

The system model, (equation 3), is a first order differential equation relating input flow rate, Q_i, to the output water level, H. In order to design a linear controller for the tank level, we must linearise the equation by considering small variations h about the normal operating level of fluid in the tank. Let,

$$H = H^o + h$$

Where, H^o is the normal operating level, and is a constant, h is a small change about that level. Then, for small variations of h about H^o, we can approximate the nonlinear function by the tangent at H^o. This allows a linear differential equation to be obtained:

$$T\frac{dh}{dt} + h = g.q_i \qquad \textbf{(4)}$$

Where q_i the variation in the input flow Q_i, needed to maintain the normal operating level H^o. The time constant T and the gain g are functions of the system parameters and the operating level.

2.2. Coupled Tank Model

When two tanks are joined together the coupled tanks system is formed (figure 2). What is the control target with the coupled tanks? In the coupled tanks the system states are the level H_1 in tank 1 and the level H_2 in tank 2. If the control input is the pump flow rate Q_i, then the variable to be controlled would normally be the second state – the level H_2, with disturbances caused by variations in the rate of flow out of the system by valve B or by changes in valve C. It is necessary to build a model for each of the tank levels

For Tank 1 the flow balance equation is:

$$Q_i - Q_b = A\frac{dH_1}{dt} \qquad \textbf{(5)}$$

where, the new variable is the flowrate Q_b of fluid from tank 1 to tank 2 through valve B.

For Tank 2 the flow balance equation is:

$$Q_b - Q_c = A\frac{dH_2}{dt} \qquad \textbf{(6)}$$

The new variable is the flowrate Q_c of fluid out of Tank 2 through Valve C.

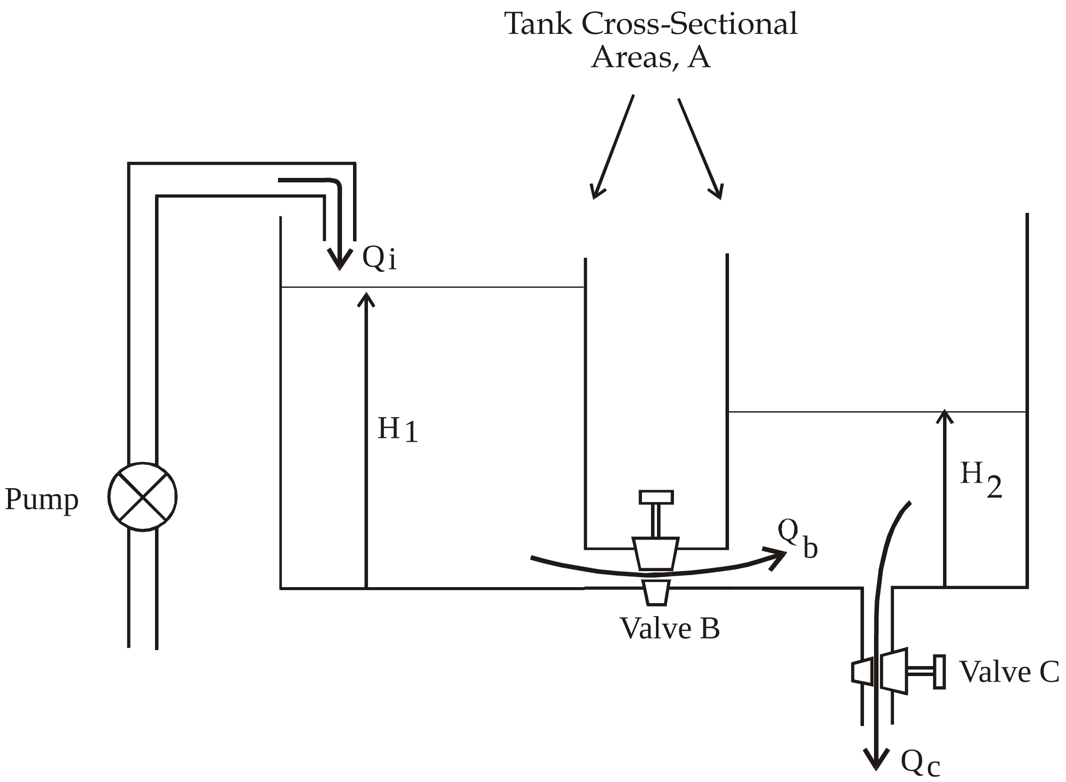

Figure 2: A Coupled Tanks System

The system model comes from the two flow balances and the nonlinear equations for flows through the valves. If the valves are ideal orifices the system nonlinearity is again a square root law. The two flow balances for ideal valves are:

$$\begin{aligned} Q_i - C_{db} a_b \sqrt{2g(H_1 - H_2)} &= A\frac{dH_1}{dt} \\ C_{db} a_b \sqrt{2g(H_1 - H_2)} - C_{dc} a_c \sqrt{2gH_2} &= A\frac{dH_2}{dt} \end{aligned} \qquad \textbf{(7)}$$

Equations 7 describe the coupled tanks system dynamics in its non-linear form with ideal equations for the valves. In general applications, the square root law is only an approximation. To design the control systems for the coupled tanks the equations are linearised by considering small variations q_i in Q_i, h_1 in H_1 and h_2 in H_2. The variations are measured with respect to the normal operating levels, H_1^o and H_2^o. Linearisation of the equations (7) gives the state equations for the coupled tanks:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} A^{-1} \\ 0 \end{bmatrix} q_i \\ \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \qquad (8)$$

The coefficients of the state matrix, $k_{11}, k_{12}, k_{21}, k_{22}$ are all functions of the operating levels in the tanks, and so the state model parameters will vary as the operating level is varied. Please to remember that the system has time varying dynamics when you think of controller design.

Transfer function models are popular in process systems and so I will also write the transfer function equation. Defining the system output to be the level in tank 2 and the input to be the pump flow, the transfer function of the coupled tanks is:

$$\frac{h_2(s)}{q_i(s)} = \frac{G}{(T_1 s + 1)(T_2 s + 1)} \qquad \textbf{(9)}$$

The time constants T_1 and T_2 are related to the operating levels in the tanks, the difference in levels in the tanks and are directly proportional to the cross sectional area of the tanks. I complained one day to my boss that I had to stay late at night because it was taking a long time to make a step test on a milk processing plant with large hold up tanks. I remembering him replying – 'Fat tanks make large time constants Elke, and large time constants can smooth out many process problems'.

2.3. Multi Input - Coupled Tank Model

The coupled tanks system can be extended in many ways. The next most interesting form is the multi-input coupled tanks. This is made with another pump supplying fluid to tank 2 and another valve allowing fluid to leave the bottom of tank 1. This makes a system with two interacting outputs (h_1, h_2) and two inputs (q_1, q_2). The result is an interesting multivariable system with many control possibilities – these are beyond the white paper scope. Maybe one day I will make a white paper on the interesting things that multivariable control can do. However, not today.

3. Example of a Tanks Control System

The figure 3 shows the CE105 Couple Tanks system. This representation of the coupled tanks control problem has become a standard system among control system educators, because it includes all the important features of level control in a simple and flexible design.

Figure 3. The CE105 Coupled Tanks System

The key features of the Coupled Tanks can be seen in the figure 3. There are two tanks placed side by side. Between them at the bottom is an adjustable valve that can be used to change the flow between the tanks. At the bottom of each tank is an adjustable valve that can be used to change the flow out of the tank. At the left hand side is an electrically controlled pump which controls the flow into the first tank. This flow is controlled by an electrical signal applied to the pump drive system from a control system. It is also possible to put another electric pump on the right hand side of the tank 2 so that there can be two externally controlled input flows to the tanks. The outputs are the measured levels in each of the tanks. You can see that the Coupled Tanks can be used to represent several different kinds of level control systems. The simplest is the single tank level control, then comes the double tank level control and the most complex is the multivariable coupled tanks process. Using the valves in different positions the parameters of the control system can also be altered to give many combinations of time constants and interactions.

Remember my story about ‘fat tanks’? Well the Coupled Tanks has been designed to give time constants in the range of 50 to 200 seconds. This range gives a good idea of the problems of controlling systems with with slow responses, but not too slow that you have to stay all night to make measurements!

4. Couple Tanks Level Control

There are many, many alternative controller design theories that can be used to control the level of fluid in tanks. All the methods that I listed out in my Servo Control white paper are possible, including fuzzy control. BUT, it is important to know that the parameters in level control system models can be hard to measure exactly. Also the square root model of flow through the valve is an approximation. Because of these uncertainties, engineers prefer to measure the time constants of the system experimentally during design and to use simple control laws that have two important properties:

- Technicians can tune the control system with not much training.
- The control law works acceptably even when the system parameters change.

Because of these requirements it is common to use the famous PID (proportional, integral, derivative) control law. In the next section I will show how this might be done for a single tanks configuration of the Coupled Tanks. To prepare for this I have closed the valve between the tanks and set the valve in the bottom of tank 1 to a middle value.

5. Example of a Level Control System Design

The specified control problem is to keep the level of the fluid in tank 1 at the mid point. The steady state pump voltage to keep the fluid at this level is approximately 3volts. I have made a step increase of 1volt to the pump voltage and measured the step response in the fluid level. From this I found that the time constant T and gain g for the first order model are approximately:

$$T = 40 \text{ seconds}$$

$$g = 1.5$$

These results are approximate because the step response is not completely steady as is shown in the figure of the step test which I made (figure 4). The level drifts during the test and so the end point is hard to determine and there is always some noise on the measured fluid height signal. These are common practical problems in real systems. The problems can be overcome with system identification tools but I want to show measurement methods that are easy to use with minimum equipment and when you must set up a controller in a short time.

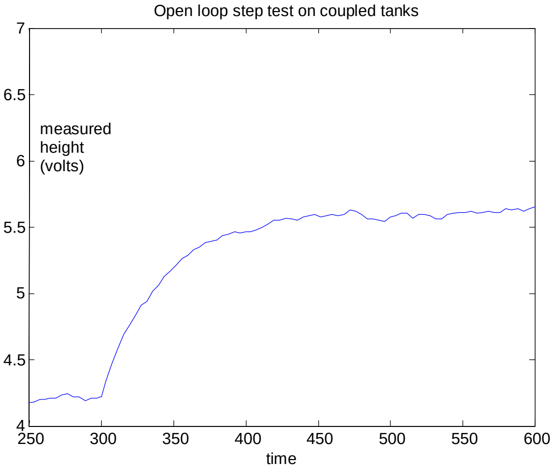

Figure 4. Step test on tank level.

Personally I do not often use derivative action in my controllers and for level control it is possible to use PI (proportional plus integral) control. The integral term is used to give zero steady state error and the proportional term allows the speed of closed loop response to be adjusted. An experienced process engineer would be able to set the PI controller after one look at the step test, and a short look at the control valve sizes in the tank feed line. However, let us do it scientifically – we can learn 'short cuts' later. The equation for a PI controller is:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau \qquad \textbf{(10)}$$

Where, k_p is the proportional gain, k_i is the integral gain, *e(t)* is the error between the reference signal *r(t)* and the output *y(t)* and *u(t)* is the signal applied to the pump drive. Taking the transfer function of equation (10) and combining it with the transfer function of the single tanks model, the closed loop equation is:

$$y(s) = \frac{g(k_i + k_p s)r(s)}{Ts^2 + s(1+gk_p) + gk_i} + \frac{(sg)d(s)}{Ts^2 + s(1+gk_p) + gk_i} \qquad \textbf{(11)}$$

This is the equation of a second order system, where *r(s)* is the reference signal and *d(s)* is a disturbance to the tank flow. We can compare parameters with the standard second order equation and pick values of proportional and integral gain to give a desired undamped natural frequency and damping factor. Here are some tips from *Tante Elke* for designing the PI controller:

- The second order system response gives the disturbance rejection part of the transfer function – this is what is normally required because the reference level is constant. The response to changes in the reference signal will depend upon the numerator for the reference signal part of the transfer function as well as the denominator.
- Do not try to 'speed up' the tanks response to much – the sizes of valves and pumps limit how fast a system can change. Demanding a fast response may saturate or damage the actuators and give nonlinear problems.
- Always check the steady state operating positions of the valves and drives in the system. The control valves should be operating between 30 and 70% for the normal operating conditions. If not there may be a valve size problem.
- Be careful of closed loop responses that give large integral gains – the integrator can 'wind-up' and cause big practical problems in the controller.

- Test your settings on a simulation such as the CE2000. Simulations can check quickly the approximate response and can save a lot of 'hand-tuning' of the real system.

From experience I know that an undamped natural frequency of 0.01 Hz and a damping factor of 1 can give good disturbance responses with not too much control action. The control system set up for this gives the controller parameters:

$$k_p = 2.7, k_i = 0.1$$

Gain values as these are very reasonable, and the control system is conservatively tuned. Larger gains may give faster responses if required. The control signals to the actuators will be larger and can have a bad impact on the valves and motors. My boss in the milk product factory had told me it was OK to tune controller to be very fast so long as I was the engineer who had to come to the factory at night to repair the valve systems. After this I have tuned my controllers so that I can sleep at night.

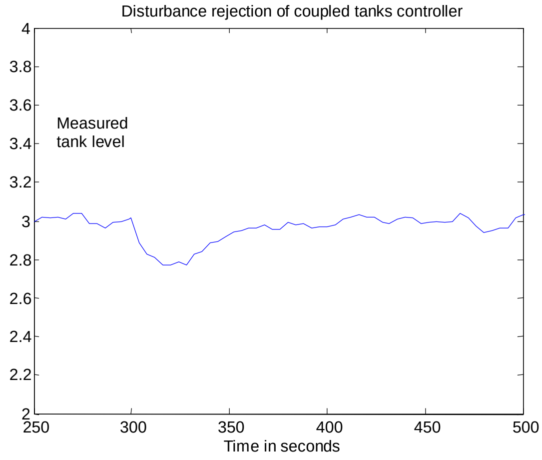

Figure 5. Disturbance rejection of the closed loop couple tanks system

Finally the figure 5 shows the disturbance rejection response for the control system. The reference signal is 3volts (about 40% capacity of the tank) and the disturbance was made by opening the output flow valve by 10%. The level recovers after about 60 seconds – this could be made faster by tuning the controller, but I would only do this if the performance specification required a faster system. Also the system controller works quite well when the reference signal is changed (this changes the time constant), so there is some robustness in the controller. I am not making a robustness analysis of this type of system normally because it is so simple – but many people do this.

6. A Final Word from Elke

I hope that you have got some ideas about the importance of coupled tank control systems from this white paper. I am sorry to say that it is not possible to answer general questions about the contents of our white papers, unless we have a contract with your organisation. For more information about the CE105 coupled tanks systems or the control simulation tool CE2000 go to the TQ Education and Training web site using the links on our web site www.control-systems-principles.co.uk or use the email info@tq.com. For more information on proportional plus integral control and three term control check the download page of our web site, we plan a white paper on three term control and it may appear at the time you look. If it is not there then be patient with us. To learn more background on level control read a control theory book. A book that we use is: *Modern Control Systems*, R.C. Dorf and R.H. Bishop, Addison Wesley.

Aufwiedersehen!

Elke Laubwald