BALL AND HOOP 2: Control and Analysis

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ABSTRACT: This is one of a series of white papers on systems modelling, analysis and control, prepared by Control Systems Principles.co.uk to give insights into important principles and processes in control. In control systems there are a number of generic systems and methods which are encountered in all areas of industry and technology. These white papers aim to explain these important systems and methods in straightforward terms. The white papers describe what makes a particular type of system/method important, how it works and then demonstrates how to control it. The control demonstrations are performed using models of real systems designed by us, and which have been developed for manufacture by TQ Education and Training Ltd in their CE range of equipment. This white paper is about the control of the Ball and Hoop system, and should be read in conjunction with the 'Ball and Hoop' white paper.

1. What is this White Paper About?

No examples of dynamic performance and controller design were given in the first ball and hoop white paper. To compensate for this omission, this further white paper has been written to give some better notes on control of the ball and hoop. This white paper concerns some of the dynamic performance features that can be demonstrated with the ball and hoop, plus a simple demonstration of slop/slosh control using dynamic feedback. A further white paper is planned that will deal with more sophisticated ways of hoop and slop control.

2. The Ball and Hoop System

From the first ball and hoop white paper, the system is an analogue of the liquid slop/slosh problem that occurs when fluids are transported, either as a cargo or a fuel. The movement of the ball in the inner periphery of the hoop reproduces the oscillations of fluids in tanks and tankers during transportation. The ball and hoop is also a good system for demonstrating some basic concepts in control and systems dynamics – such as system zeros and non-minimum phase systems. Figure 1 shows the CE109 system.

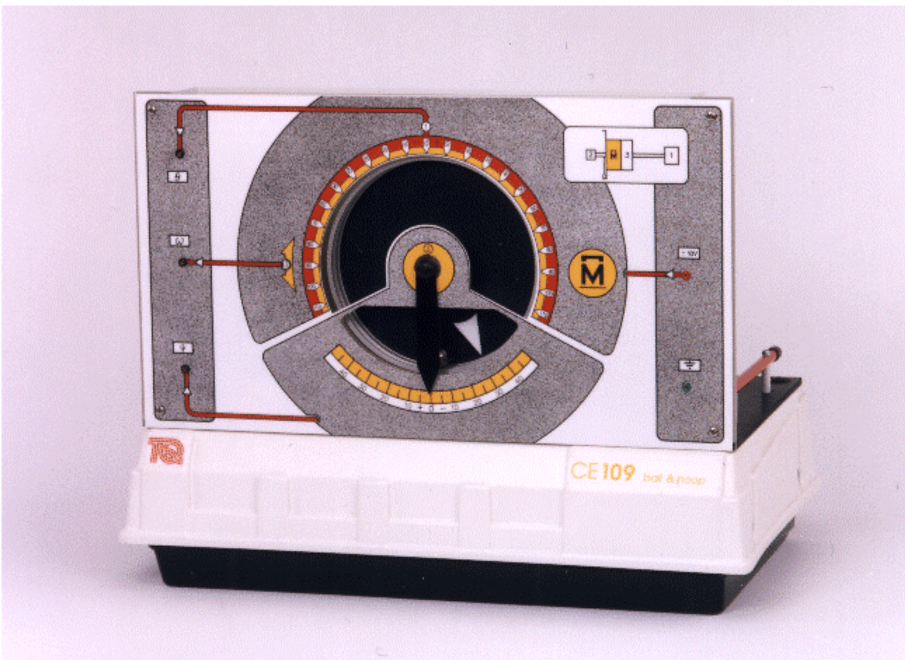


Figure 1. The CE109 Ball and Hoop System

The main hardware elements are:

1. A hoop which can rotate with a steel ball on its inner periphery. (The black disc in the centre of the figure is the hoop.
2. The servomotor, M, which drives the hoop and controls the hoop angle via the motor torque, $\tau(t)$.
3. A sensor of the hoop angle, θ. (The white arrow painted on the black disc is a visual indication of hoop angle.
4. A sensor for the ball angle in the hoop, i.e. the slop angle ψ. (The black pointer pointing vertically down is a visual indication of ball angle).

3. The Ball and Hoop System Model

The Ball and Hoop model and dynamics were derived in the first Ball and Hoop white paper. Here only the terminology and final models are discussed.

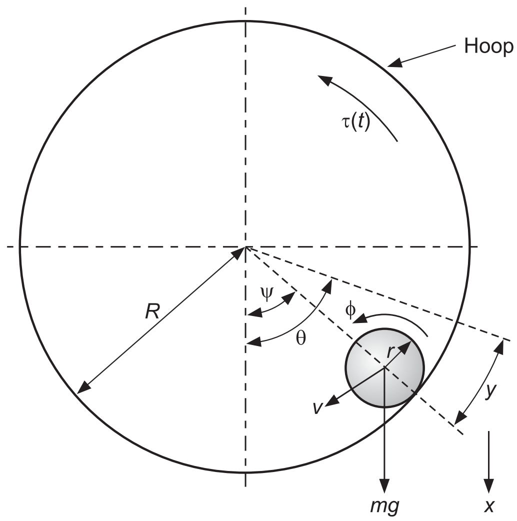


Figure 2. The Ball and Hoop Schematic

Figure 2 shows the ball and hoop schematic, from which the key system variables are:

The hoop radius: R.
The ball radius: r.
The ball mass: m.
The hoop angle, θ
The ball angle with the vertical, (the slop/slosh angle): ψ.
The ball position on the hoop, y.
The input torque to the hoop, $\tau(t)$

The differential equations of motion are:

$$\begin{pmatrix} I_a + m(R-r)^2 & \dfrac{-m(R-r)^2}{R} \\ \dfrac{-m(R-r)^2}{R} & \dfrac{I_b}{r^2} + \dfrac{m(R-r)^2}{R^2} \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} b_m & 0 \\ 0 & \dfrac{b_b}{r^2} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{y} \end{pmatrix} + mg \begin{pmatrix} (R-r)\sin(\theta - \dfrac{y}{R}) \\ -\left(\dfrac{R-r}{R}\right)\sin(\theta - \dfrac{y}{R}) \end{pmatrix} = \begin{pmatrix} \tau(t) \\ 0 \end{pmatrix}$$

(1)

Linearized Differential Equation Model

By assuming that the slop angle $\psi = \left(\theta - \frac{y}{R}\right)$ is relatively small, we can replace $\sin\left(\theta - \frac{y}{R}\right)$ by $\left(\theta - \frac{y}{R}\right)$ to get a linear second order matrix equation of the form:

$$\begin{pmatrix} I_a + m(R-r)^2 & \frac{-m(R-r)^2}{R} \\ \frac{-m(R-r)^2}{R} & \frac{I_b}{r^2} + \frac{m(R-r)^2}{R^2} \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} b_m & 0 \\ 0 & \frac{b_b}{r^2} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{y} \end{pmatrix} + mg \begin{pmatrix} (R-r) & -\left(\frac{R-r}{R}\right) \\ -\left(\frac{R-r}{R}\right) & \frac{R-r}{R^2} \end{pmatrix} \begin{pmatrix} \theta \\ y \end{pmatrix} = \begin{pmatrix} \tau(t) \\ 0 \end{pmatrix} \quad \textbf{(2)}$$

Linear State Space Model

The equation (2) is a specific form of the equation:

$$M\ddot{X} + D\dot{X} + KX = Bu \quad \textbf{(3)}$$

This general form is often used in robotics, [1] to neatly formulate the differential equations of a system. It is also a convenient way to formulate state space equations for the system. Specifically by putting $X = (\theta \quad y)^T$ and $X_1 = X,\ X_2 = \dot{X}$, then the state equations from Equation (3) are:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} u$$
$$Y = C \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad \textbf{(4)}$$

The choice of the 'read out' matrix C will depend upon the measured states.

Approximations and Substitutions

Because the hoop radius is much bigger than the ball radius, e.g. $R >> r$, then Equation (2) can be written:

$$\begin{pmatrix} I_a + mR^2 & -mR \\ -mR & \frac{I_b}{r^2} + m \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} b_m & 0 \\ 0 & \frac{b_b}{r^2} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{y} \end{pmatrix} + mg \begin{pmatrix} R & -1 \\ -1 & \frac{1}{R} \end{pmatrix} \begin{pmatrix} \theta \\ y \end{pmatrix} = \begin{pmatrix} \tau(t) \\ 0 \end{pmatrix} \quad \textbf{(5)}$$

Also the moment of inertia of a solid ball is $I_b = \frac{2}{5} m r_b^2$, (note the ball radius and rolling radius are different) so that the equations become:

$$\begin{pmatrix} I_a + mR^2 & -mR \\ -mR & m\left(\frac{2r_b^2}{5r^2} + 1\right) \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} b_m & 0 \\ 0 & \frac{b_b}{r^2} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{y} \end{pmatrix} + mg \begin{pmatrix} R & -1 \\ -1 & \frac{1}{R} \end{pmatrix} \begin{pmatrix} \theta \\ y \end{pmatrix} = \begin{pmatrix} \tau(t) \\ 0 \end{pmatrix} \quad \textbf{(6)}$$

Transfer Function Model

The matrix differential equations can sometimes be separated. In particular, if the inertial torque of the ball, is small compared to that of the hoop and the motor torque, then the first row in eqn. (6) becomes the well known differential equation for a DC motor with an inertial load, I_a, and viscous friction b_m:

$$I_a\ddot{\theta} + b_m\dot{\theta} = \tau(t) \tag{7}$$

and the second equation in eqn. (6) is:

$$\left(\frac{2r_b^2}{5r^2}+1\right)\ddot{y} + \frac{b_b}{mr^2}\dot{y} + \frac{g}{R}y = R\left(\ddot{\theta} + \frac{g}{R}\theta\right) \tag{8}$$

These two separate differential equations are useful because they let us write the system model as two cascaded transfer functions, where in the first transfer function the motor torque produces a hoop angle, and in the second transfer function the hoop angle produces a ball position. Again this can be useful in setting up a simulation model and designing controllers.

4. Special Features of the Ball and Hoop System Dynamics

Two special features of the ball and hoop are the ability to demontrate 'zeros of transmission' and to show 'non-minimum phase behaviour'.

Zeros of Transmission

Writing equation (8) as a transfer function, thus:

$$\frac{y(s)}{\theta(s)} = R\left[\frac{s^2 + \frac{g}{R}}{\left(\frac{2r_b^2}{5r^2}+1\right)s^2 + \frac{b_b}{mr^2}s + \frac{g}{R}}\right] \tag{9}$$

shows that the transfer function from the hoop angle, $\theta(s)$, to the ball position on the hoop, $y(s)$, has a pair of imaginary zeroes at $s = \pm j\frac{g}{R}$. These are 'zeros of transmission' and it means that when the hoop angle is a sinewave of this frequency then the ball position output is exactly zero. The model of the ball and hoop can be used to illustrate this zero of transmission by implementing a simulation, (we show the real thing with the real equipment later in this white paper).

Using the Matlab model given at the end of this white paper, the zero can be demonstrated. In particular, Figure 3 shows results of a simulation with the hoop angle under position control and the input to the position controller is a sine wave of frequency 1.8Hz, (this approximately corresponds to the zero of the transfer function Eqn. (9)). The ball position *y(t),* the slop angle, *ψ(t)* and the hoop angle *θ(t)* are shown. Note that after the initial conditions, the steady state signal *y(t)* decays to almost zero, whereas the other signals are remain large. Compare this with Figure 4 which shows the corresponding simulated signals when the hoop position controller reference input is a sinewave of frequency 1.5Hz. The large size of the hoop position signal is due to the resonance of the transfer function Eqn (9) in the region of 1.5Hz. The closely coupled zero/pole pair in the transfer function (Eqn. (9)) is very interesting for the dynamical systems analyst. A number of mechanical structures have this characteristic, and in filter theory it is a very desireable feature when transmission channels are closely packed and cross-talk levels must be low.

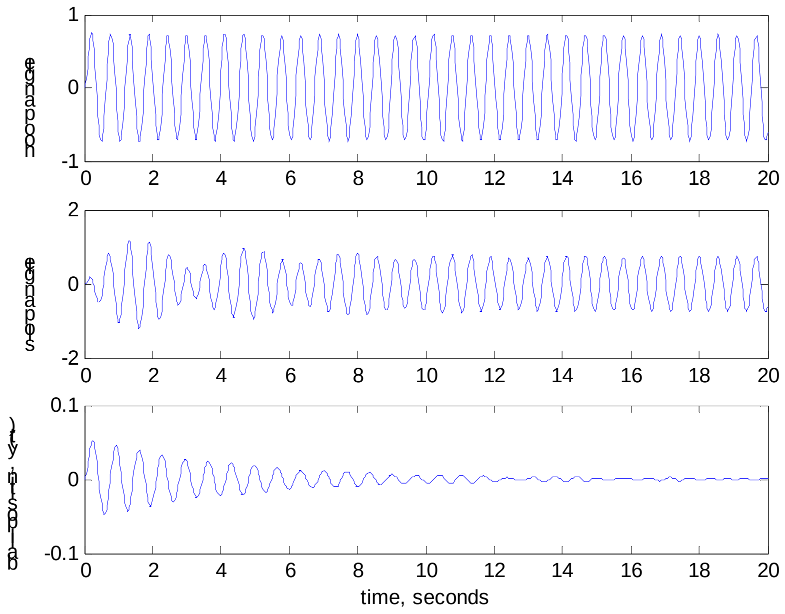


Figure 3: Simulated Ball and Hoop Outputs for a Sinusoidal Excitation at 1.8Hz

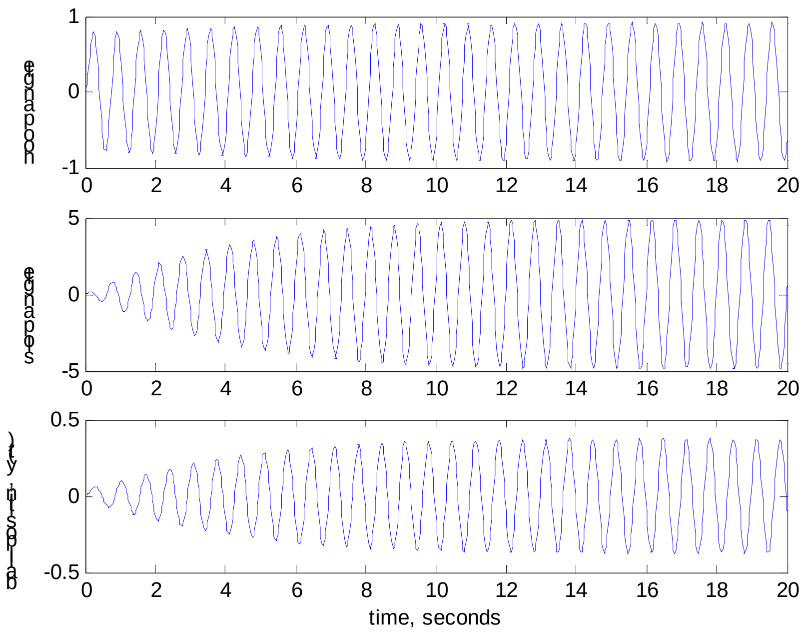


Figure 4: Simulated Ball and Hoop Outputs for a Sinusoidal Excitation at 1.5Hz

Non-Minimum Phase Behavior and Shifting Zeroes

The outputs for the Ball and Hoop are $\theta(s)$ and $\psi(s)$. From these it is possible to construct the signal $y(s)$ by subtracting scaled measurements of the hoop and slop angles. Generally, we can consider the synthetic output signal $x(s)$ is given by:

$$x(s) = \theta(s) - k_s \psi(s) \tag{10}$$

where:

k_s = Scalar gain factor.

Note that when k_s is unity, $x(s)$ is the variable $\frac{y(s)}{R}$, and this corresponds to a scaled version of the ball position on the periphery of the hoop. Note that Equation (10) is the combination of two system output signals, such that the input has two paths to the output $x(s)$. Rewriting Equation (10) as a transfer function gives:

$$\frac{x(s)}{\theta(s)} = 1 - k_s \frac{\psi(s)}{\theta(s)}$$

By varying the gain, k_s, a root locus of the transfer function zeroes can be plotted and for gains greater than unity the root locus is in the right hand plane. This means that non-minimum phase behaviour occurs in the system. This will give the characteristic feature of non-minimum phase where the output goes initially in the 'wrong' direction when step inputs are applied. By using the model of the ball and hoop to get a Matlab simulation, Figure 5 shows how the non-minimum phase characteristic in the step response grows as k_s is increased.

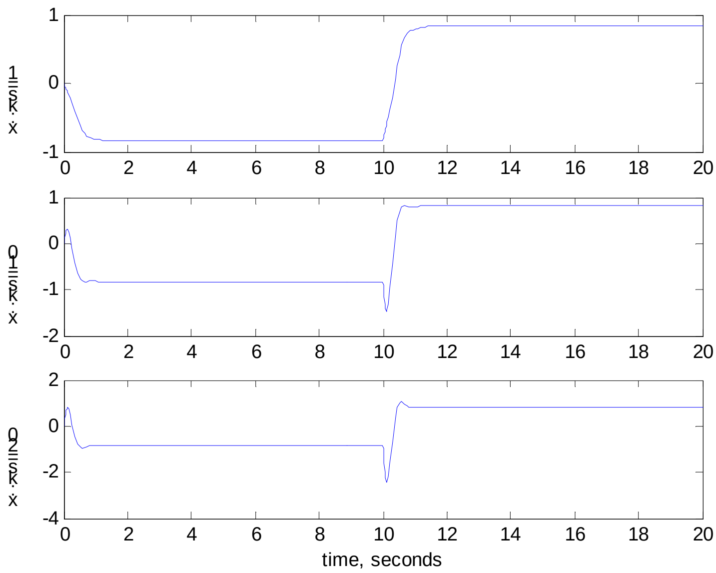


Figure 5. Simulated Non Minimum Phase Step Responses for the Signal x (see eqn. 10).

5. Slop Control

When a step change in desired hoop angle is made the ball can oscillate before settling to its new rest position – e.g. it exhibits slop. By feeding back a component, k1, of the slop angle $\psi(s)$, it is possible to suppress the ball oscillations and thus perform a dynamic form of slop control. Again using the simulation, this is illustrated in Figure 6, which shows the slop angle with and without slop angle feedback to the hoop position controller.

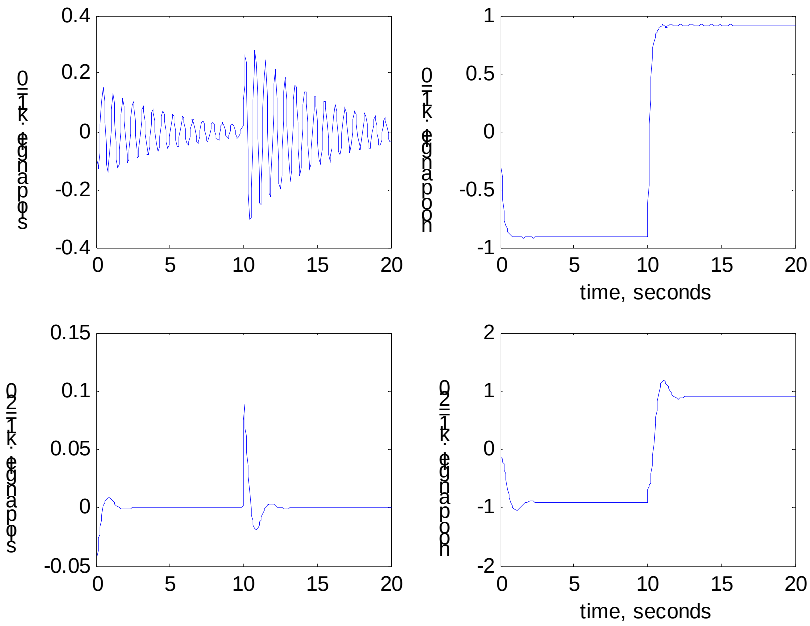


Figure 6. Results of Slop Angle Feedback (Simulation)

6. Experimental Results using the CE 109 Ball and Hoop System

In this section we present some experimental results which extend the simulations presented in the previous sections. These real life experiments are very important, simulations are not the real thing. The experiments use a CE109 Ball and Hoop System controlled using the CE2000 Real Time Software and the interface unit CE122. No other special equipment is required. Check our website (www.control-systems-principles.co.uk) in case we have video clips for this experiment.

The first step is to design a position control feedback loop for the hoop. To do this we can use the nominal linear model shown in the block diagram below (Figure 7). The speed and position gains are adjusted to give a suitable step response for the hoop position. Note that this model is essentially experimental. It neglects the motor electrical time constant and other uncertainties in the motor/hoop dynamics. The various constants in the model are obtained experimentally for each Ball and Hoop system. The block diagram indicates the maximum values of the signals and their units.

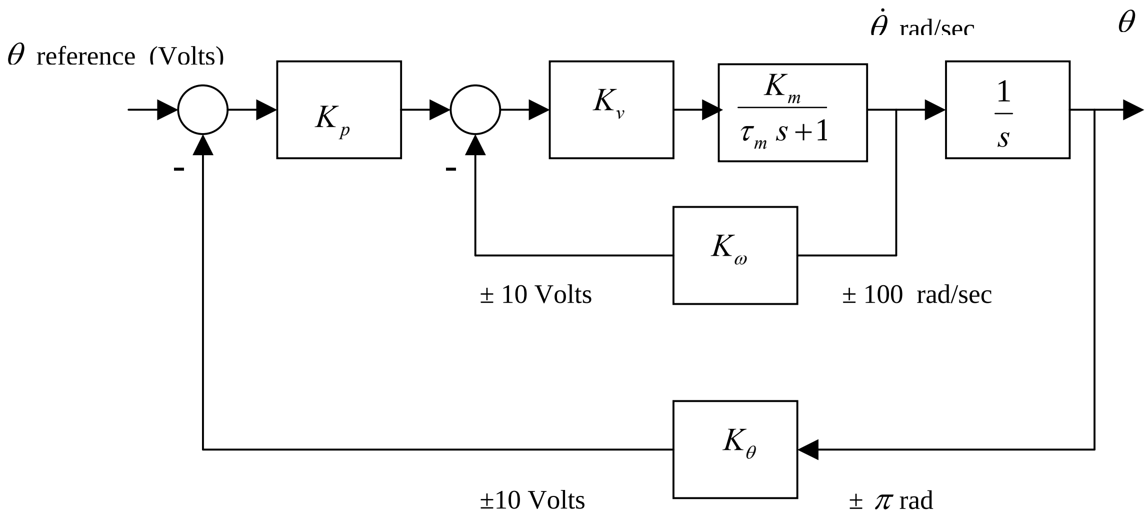


Figure 7. Hoop Angle Control

The parameters used in the figure are:

K_p = Position controller gain (0.2)

K_v = Velocity controller gain (15)

K_m = Servomotor gain (30 rad/s/Volt)

τ_m = Servomotor time constant (1.0 Sec)

K_ω = Velocity scaling (0.03 Volts/rad/sec)

K_θ = Position scaling (5.73 Volts/rad)

K_ψ = Slop angle sensor gain (14.32 Volts/rad or 0.25 Volts/deg)

The nominal values of the parameters, as determined from our experimental set up, are given in brackets.

Choosing K_p and K_v.

K_v and K_p are chosen so that the closed-loop hoop position transfer function is of the form,

$$T(s) = \frac{\theta(s)}{R(s)} = \frac{\omega_n^{\,2}}{s^2 + 2\zeta\,\omega_n s + \omega_n^{\,2}}$$

where

$$\omega_n^{\,2} = \frac{K_p\,K_m\,K_v\,K_\theta}{\tau}$$

and

$$2\zeta\,\omega_n = \frac{1 + K_m\,K_v\,K_\omega}{\tau}$$

From the values above the nominal closed-loop dynamics can be obtained. For example say we require $\omega_{\mathrm{n}} = 9$ and $\zeta = 1$ then $K_p = 0.028$ and $K_v = 17$. Note that actual values will be slightly different for each ball and hoop and the above analysis assumes continuous time control. However with a sampling time Ts=50ms we are well above the closed-loop bandwidth of the control system. After the position control system has been tested transmission zeros, resonance and slop control can be demonstrated.

Transmission Zeros.

The normalized transfer function from hoop angle to the signal y(t) (Equation 9) with the ball and hoop model parameter measured from our ball and hoop system (given in the Matlab model), is

$$\frac{y(s)}{\theta(s)} = \frac{0.665(s^2 + 129.2)}{s^2 + 0.512s + 86.07}$$

The magnitude plot for this transfer function is plotted below in Figure 8. This plot shows a transmission zero occuring at 1.8Hz and a resonant peak at 1.47 Hz. Note that the zero frequency is slightly larger that the natural frequency of the ball in the hoop as predicted by Equation 9.

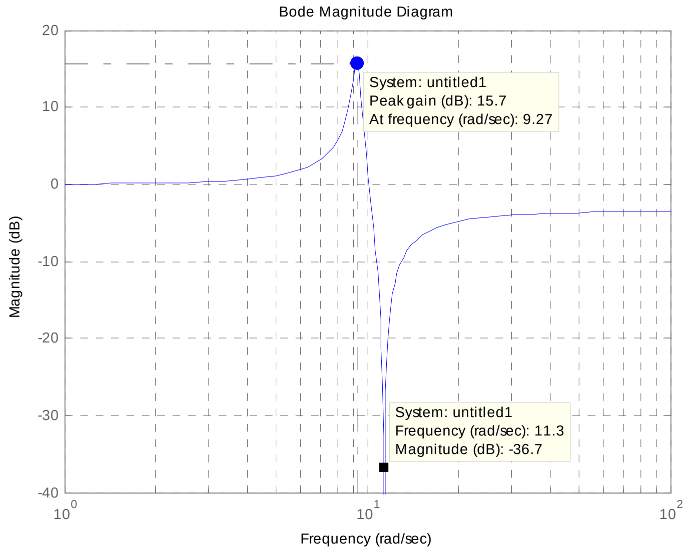


Figure 8. Frequency Response Gain Plot of Equation 9, using Actual Ball and Hoop Parameters, and showing the Transmission Zero and Resonance

At the transmission zero the steady state output of y(t) to a sinusiod will be zero. Physically this occurs when there is no relative motion between the ball and hoop which can be observed visually. The hoop position reference is set to a 1.8 Hz sinewave with an amplitude of 1.0 and the system is allowed to reach steady state. In a practical system, the reference frequency must be tuned slightly to find the exact zero frequency. At the frequency of the transmission zero the ball and hoop will be synchronised. Occasionally, due to external disturbances, a transient will occur causing the the ball and hoop to temporarily go out of synchronisation. After some time the ball and hoop should return to synchronization. The signal

$$y(t) = \theta(t) - k_s \psi(t)$$

described in the text is constructed from the hoop and slop angle sensor outputs. When the system is tuned correctly the slop angle and hoop angle voltages will be a constant amplitude. In the CE109 Ball and Hoop, the slop and hoop angle voltages have DC components. These are subtracted out before the signal *y(t)* is formed. The signals are displayed on the CE2000 chart recorder and zeroed visually. To find the correct value of k we notice that when *y(t)* = 0 then $\theta(t) = k_s \psi(t)$. This allows us to find an approximate value of k. For our ball and hoop k = 0.225. The transient response to a sinusiodal input is shown in Figure 9a. Just as in the simulation the output decays to zero as the ball and hoop are synchronized. At first glance this result is counter intuitive. It might be expected that this zero should occur at the ball natural frequency – so here's an open question. Why are the hoop and ball oscillations synchronised at a frequency larger than the ball natural frequency of oscillation?

Resonance

The hoop angle reference frequency is now reduced to 1.5Hz. It may also necessary to reduce the amplitude to 0.5 volts. This prevents the amplitude of the ball becoming too large. We are driving an underdamped dynamic system close to its resonant frequency! When the steady state has been reached the ball and hoop angles will be a constant amplitude. Now however the ball and hoop will no longer be synchronized. The plot of y(t) in this case is shown in Figure 9 b.

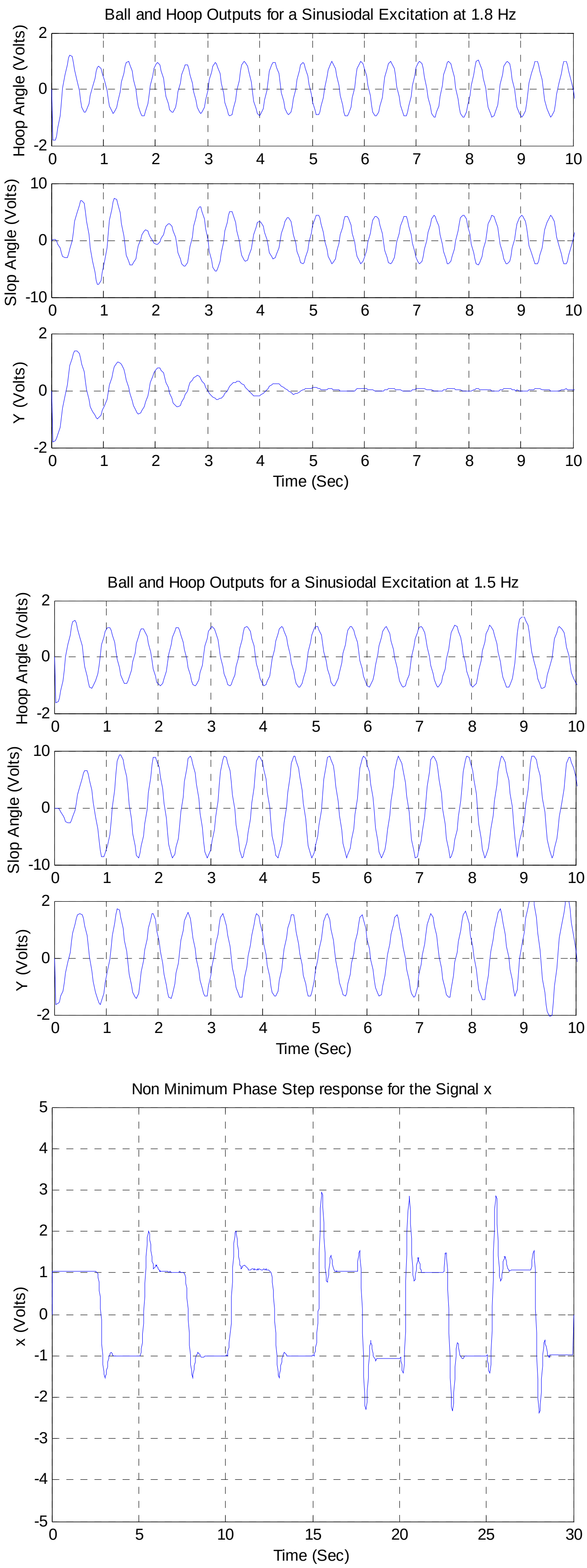


Figure 9. a) Transmission zero 1.8 Hz: b) Resonant Pole 1.5 Hz: c) Non Minimum Phase Response

Non Minimum Phase Zeros

Equation 10, repeated here, is

$$x(s) = \theta(s) - k_s \psi(s)$$

By varying the value of k_s non minimum phase behaviour is demonstrated. This is plotted in Figure 9c. Here the vaue of k_s is increased showing the characteristic response of a system with nonminimum pahse zeros.

Slop Control: A classic engineering tradeoff

To demonstrate slop control we first place a position feedback loop around the hoop. Now additionally the slop angle is fed back using a proportional controller. To explain why this works consider the following feedback system,

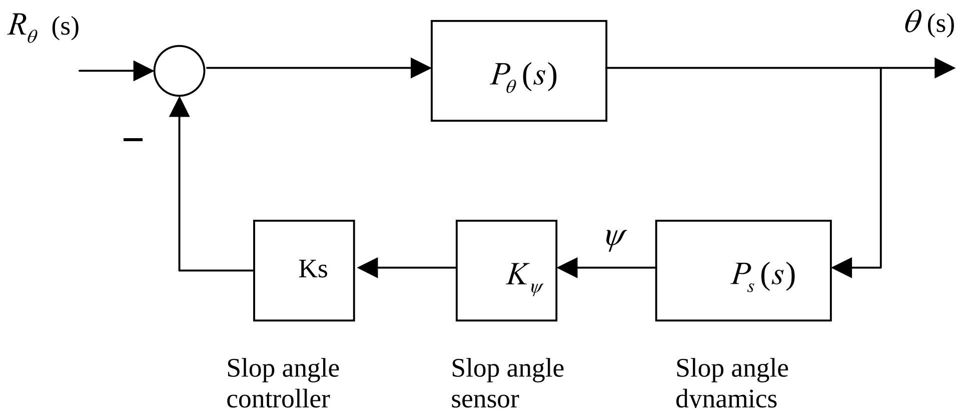


Figure 10. Slop Angle Control System

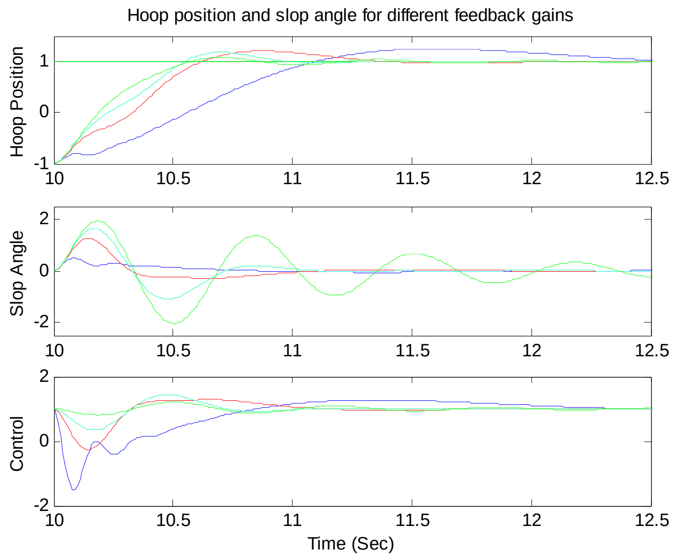
where

$$P_\theta(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\,\omega_n s + \omega_n^{\ 2}}$$

$$P_s(s) = \frac{0.334s(s+1.532)}{s^2 + 0.667s + 87}$$

and Ks is the slop controller gain. The forward path in the above feedback system is just the closed-loop hoop angle dynamics. The hoop angle in turn excites the slop dynamics. The slop angle is fed back via a negative feedback loop to the hoop angle reference signal. Note that the steady state slop angle is zero so that the hoop angle will track a constant reference. Next we have to choose the slop angle feedback gain. This is a classic engineering trade-off. The hoop angle is required to respond rapidly to a step command without causing large movement in the ball position. Note in this context the Ball and Hoop can be thought of as a single input two output system (SITO). With the available control we would like to move the hoop fast and at the same time minimize the ball deviation from zero.

We are allowed to choose the closed-loop hoop dynamics and the slop angle controller gain. Simulations of the above system suggest choosing ξ = 1.0 and ω_n = ball natural frequency. This places the dominant closed-loop poles of the hoop dynamics on real axis at $\pm\omega_n$. If Ks is zero there will be no slop control signal and the ball motion is not damped. The closed-loop hoop angle will be unaffected. However as Ks is increased the hoop will be forced to move more and more slowly to maintain a small slop angle deviation. This can be understood by examining the root locus of the above system as the controller gain Ks is varied. We then choose the control gain so that the real parts of two sets of complex poles are the same. For larger gains the hoop dynamics dominate the closed-loop response. For smaller gains the ball dynamics dominate the closed-loop response. This is illustrated in the simulation and root locus below (Figures 11 and 12). For a small controller gain the ball is only lightly damped while the hoop response is fast. For large values of controller gain the hoop position is sluggish. So an intermediate value of controller gain has to be selected as a compromize between the two extremes. A control gain of



K=0.4 is a good compromise giving the closed-loop poles as indicitated below on the root locus.

Figure 11. Slop Angle Control Simulation

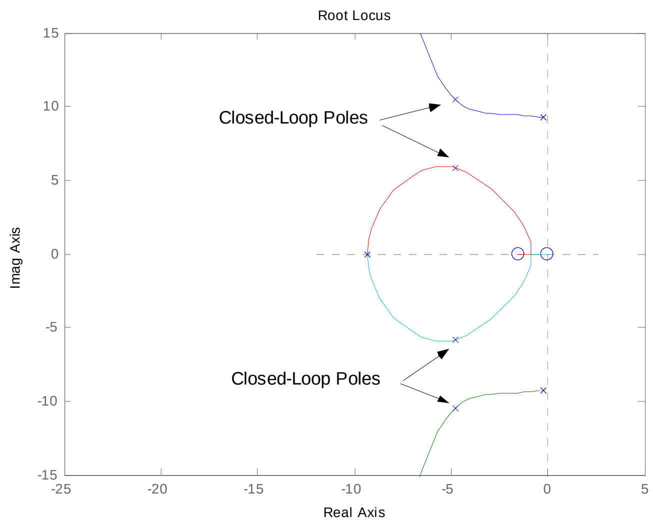


Figure 12. Root Locus: Optimal Position of Closed-Loop Poles.

The experimental results for a CE109 Ball and Hoop are shown below in Figure 13. Clearly slop control has a significant effect on ball damping. The hoop angle also responds slower, just as predicted by theory.

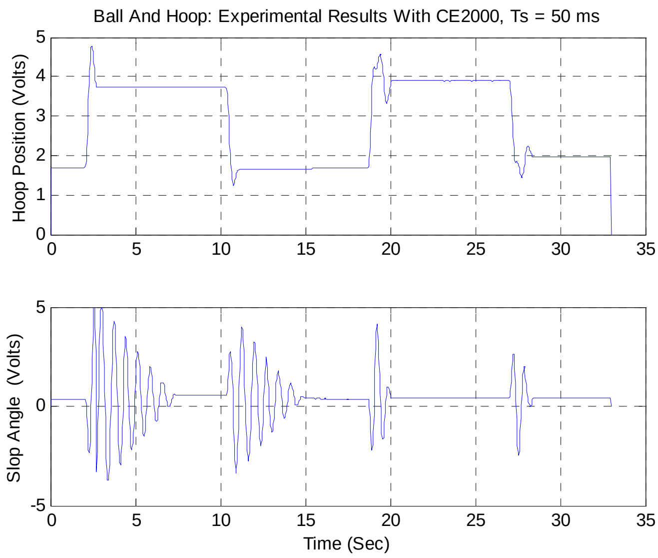


Figure 13. Experimental Results Showing Increasing Values of Proportional Slop Control.

Closed-loop dynamics

The closed-loop dynamics from hoop position reference to hoop position are given by

$$\frac{\theta(s)}{R_\theta(s)} = \frac{88.83(s^2 + 0.507s + 86.34)}{s^2 + 9.673s + 59.64)(s^2 + 9.648s + 128.6)}$$

from which we find that the closed-loop zeros are the poles of the slop angle dynamics and that the closed-loop poles have approximately the same damping term as predicted in the above analysis. The closed-loop slop dynamics have the same closed-lop poles but now the closed-loop zeros are the same as the slop-angle open-loop zeros.

$$\frac{\psi(s)}{R_\theta(s)} = \frac{29.67s(s + 1.532)}{s^2 + 9.673s + 59.64)(s^2 + 9.648s + 128.6)}$$

The CE2000 controller used to obtain the experimental results is shown below in Figure 14. The sampling time Ts=50ms .

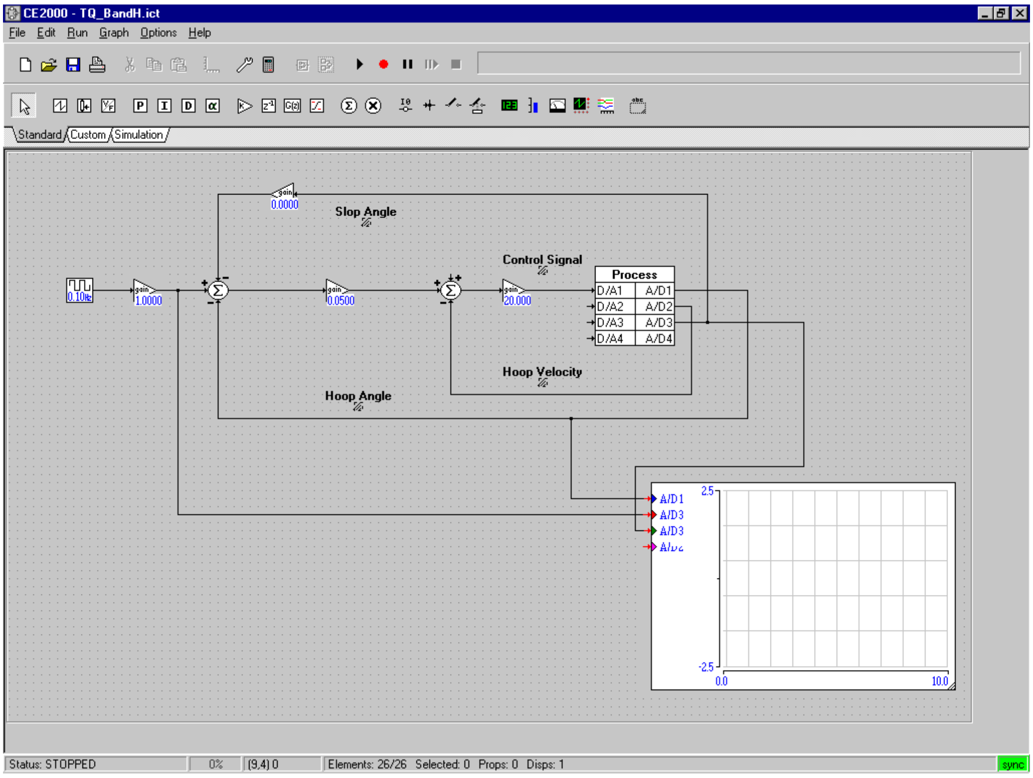


Figure 14. CE2000 Controller for Experimental Work

7. Simulation and Design Software

7.1. Matlab Code for the Ball and Hoop Model

```
%
 % Define the ball and hoop model parameters for a CE109 Ball and Hoop system
 %
 bm = 0.1;        % estimate of the friction coefficient for the rotation of hoop
 bb = 1.67e-6;    % measured
 Ia = 1.6e-3;     %Ia is M*(R^2+Ri^2)/2 and can be calculated from the hoop outer radius (R)
                  % and inner radius (Ri)  and the mass of the hoop (M). (approximately) 0.1kg

 R = 0.085;        % 20cm diameter hoop (approximate)
 r = 0.0091;      %2cm diameter ball   (approximate)
 m = 0.032;       % mass of the ball   (approximate)
 Ib = 2*m*r^2/5; %Ib is calculated from 2*m*r*r/5 using radius of ball (r) and the mass of the ball (m)

 g = 9.81;        % gravitational acceleration
 %
 % Define the state model - full equations
 %
 M = [Ia+m*(R-r)^2, -m*(R-r)^2/R;
     -m*(R-r)^2/R, Ib/r^2 + m*((R-r)/R)^2];
 D = [bm, 0;
     0, bb/r^2];
 K = m*g*[(R-r), -(R-r)/R;
         -(R-r)/R, (R-r)/R^2];
 B = [1; 0];
%
% Form the state equations
```

```
%
 a = [zeros(2,2), eye(2,2); -inv(M)*K, -inv(M)*D];  %see Ball and Hoop white paper
 b = [zeros(2,1); inv(M)*B];                         %see Ball and Hoop white paper
 c = eye(4,4);    % the model outputs are all the system states (N.B. the slop velocity is not available on the real
equipment)
 d = zeros(4,1); % the input vector u is motor torque;
 %
 % Form the Ball and Hoop state space object and use it in matlab or simulink models
 %
 BandHModel=ss(a,b,c,d);
```

7.2. Matlab Code used to compute Kp and Kv

```
%
%M-file to compute Kp and Kv for Ball and Hoop (Continuous time)
%Nominal Sensor gains
%Open-loop motor velocity gain
Km=1/0.031;
%Position sensor gain
Kt=5.72;
 %Velocity sensor gain
Kw=0.032;%
Slop angle sensor gain
Kslop=14.32;
%Nominal Motor time constant
tau=1;
%
%Desired Closed-loop response
wn=2*pi*1.5;
zet=1;
%Position and velocity gains
Kv=(2*zet*wn*tau-1)/(Km*Kw);
Kp=tau*wn*wn/(Km*Kv*Kt);
['Kp=' num2str(Kp),'   Kv=' num2str(Kv)]

s=tf('s');
Tloop=wn^2/(s^2+2*zet*wn*s+wn^2); %the closed-loop hoop dynamics
```

8. A Final Word

The Ball and Hoop is one of our proudest creations and has given us a lot of fun and insights into engineering dynamics. We hope that you share our pleasure and come to understand more clearly some of the less well travelled roads of control systems engineering.

Thank you to all the students and engineers who have written in appreciation of the white papers and their experiences with the CE teaching equipment. Sadly, however we must repeat that it is not possible to answer general questions about the contents of our white papers, unless we have an arrangement with your organisation. For more information about the CE 109 Ball and Hoop go to the TQ Education and Training Ltd web site using the links on our web site www.control-systems-principles.co.uk or use the email info@tq.com. We do not have a book reference for ball and hoop control – that's why these white papers are as detailed as they are– but try our references and keep checking our website for more ball and hoop papers and the video clips that are in preparation.

9. References

[1] Readman, M. C., *Flexible Joint Robots*, CRC Press 1994

[2] Wellstead, P. E., The Ball and Hoop System, *Automatica*, Vol. 19, No. 4, pp. 401-406, 1983

[3] Tan, H., Chang, J., Chaffee, M. A., *Practical Motion Control Modelling and PI Design,* Proc ACC, Chicago, June 2000